

# A Review of Optimal Placement of Phasor Measurement Units in Power System Distribution State Estimation

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**Abstract**—Phasor measurement units (PMUs) are increasingly becoming a standard feature in substations, enabling synchronized data gathering for various applications, including system condition monitoring during normal operation and ensuring power system protection and control in unusual situations. The optimal PMU placement (OPP) problem seeks to determine the minimum number of PMUs required to achieve full observability of the system. Numerous methods have been proposed to address the OPP problem, encompassing heuristic, meta-heuristic, and mathematical programming optimization strategies. This paper provides a thorough literature review of significant contributions in this expansive area, focusing on the OPP problem and the various techniques for its solution.

**Index Terms**—About four key words or phrases in alphabetical order, separated by commas.

## I. INTRODUCTION

The reliable operation of power systems requires diligent observation of operational conditions. Data collected from various substations is utilized in control centers to assess both metered and unmetered electrical values, as well as network parameters, while also detecting and rectifying measurement and topological errors. In the past, metrics were sourced from SCADA systems, which included flows of active and reactive power, injections, and voltages at bus junctions. The integration of global positioning system (GPS) technology and sophisticated sampled data processing methods for computer relaying led to the creation of phasor measurement units (PMU). PMUs are sophisticated monitoring instruments that deliver accurate, time-stamped positive sequence measurements [1].

A Phasor Measurement Unit (PMU) installed on a bus can directly gauge both the voltage phasor and the current phasors for some or all of the interconnected branches, given that the PMU has sufficient channels. With the growing implementation of PMUs in recent years, enhanced monitoring, protection, and control of power networks have become achievable [2], [3], and [4]. However, the specific applications of PMUs, along with their relatively high unit costs and the potentially greater expenses associated with communication infrastructure, present significant challenges

in determining the optimal placement of these devices.

To tackle the OPP challenge, various traditional optimization techniques have been suggested, such as combinatorial optimization, dynamic programming, linear programming, and nonlinear programming. To overcome the limitations of traditional methods—like the risk of getting caught in local optima, difficulties in managing constraints, and numerical challenges—advanced heuristic and modern metaheuristic optimization approaches have been introduced. The techniques examined include depth-first search (DeFS), minimum spanning tree (MST), simulated annealing (SA), tabu search (TS), genetic algorithms (GA), differential evolution (DE), immune algorithms (IA), particle swarm optimization (PSO), and ant colony optimization (ACO), among others found in the OPP literature. This paper reviews the existing research on the most frequently used conventional, heuristic, and metaheuristic optimization methods to address the standard OPP problem. Section II outlines the problem formulation, while Sections III, IV, and V offer solutions to the OPP challenge using mathematical programming, heuristics, and metaheuristics, respectively. The paper wraps up with conclusions in Section VI.

## II. FORMULATING THE OPTIMAL PMU PLACEMENT ISSUE

A PMU can measure both the voltage phasor of the installed bus and the current phasors of some or all of its connecting lines. For the placement of PMUs, the following principles can be utilized [5]:

Principle 1: Assign one voltage measurement and one current measurement for each branch linked to a bus that has a PMU.

Principle 2: Provide a voltage pseudo-measurement to each node that is reached by another bus with a PMU.

Principle 3: Allocate a single current pseudo-measurement to each branch that connects two buses with known voltages. This allows for the linkage of the observed areas.

principle 4: Assign a current pseudo-measurement to each branch so that the current can be indirectly calculated using Kirchhoff's current law (KCL).

When the current balance of a node is determined, this principle can be applied. Kirchhoff's Current Law (KCL)

allows for the calculation of the current phasor in the unknown branch, provided all other branches are known. Due to (i) the extensive nature of the Optimal Placement Problem (OPP) in combinatorial optimization and (ii) the dependency of system observability on two factors—the count of PMUs and the set of placements—it is difficult to directly identify the optimal solution. The OPP features an objective function that is nonconvex, non-smooth, and nondifferentiable, presenting significant computational complexities characterized by nonlinearity, discontinuities, and multiple modes. To determine suitable locations for PMU installations, the following observability criteria must be satisfied: [6], [7]

Condition 1: The current phasors of all incident branches and the bus voltage phasor are available for PMUs installed at the buses. Condition 2: The voltage phasor at the opposite end of a branch can be derived if the voltage phasor at one end and the branch's current phasor are known. Condition 3: If the voltage phasors at both ends of a branch are known, the current phasor for that branch can be directly calculated. Condition 4: Kirchhoff's Current Law (KCL) can be applied to ascertain the current phasor of an unknown branch if there is a zero-injection bus lacking a PMU and the current phasors for all incident branches, except one, are known. Condition 5: Node voltage equations can be employed to find the voltage phasor of a zero-injection bus when its voltage phasor is unknown but the voltage phasors of all surrounding buses are accessible. Condition 6: Node voltage equations can also be utilized to compute the voltage phasors of zero-injection buses if the voltage phasors of a set of adjacent zero-injection buses are unknown, yet the voltage phasors of every other bus within that group are available.

Direct measurements are the measures that were taken from Condition 1. Pseudo-measurement is another term for the measurements derived from Conditions 2-3. Extension-measures are the measurements derived from Conditions 4-6.

### III. METHODS OF MATHEMATICAL PROGRAMMING

1. Integer linear programming (ILP) refers to a specific type of linear programming problem where all decision variables must be integers. The objective of this approach [8] is to minimize the number of phasor measurement units (PMUs) strategically placed to mitigate measurement criticality across the system. The placement issue is further broadened to consider standard measures as potential candidates for implementation. Furthermore, the same framework can be employed to determine optimal locations that factor in the desired level of local redundancy. This facilitates the development of measuring devices with varying degrees of sensitivity, thereby reducing the risk of data inaccuracies and measurement loss.
2. Greedy Algorithm: The greedy algorithm is a combinatorial optimization technique that finds the best local, or immediate, solution while doing so. A pre-processing technique for virtual data removal and The use of matrix reduction algorithms has reduced the the placement model's size and the amount of work required to identifying the best possible placement set

[9].

### IV. HEURISTIC METHODS

1. Depth First Search (DeFS): In a directed graph, the depth first search algorithm (DeFS) partitions the graph into a forest and marks all vertices in the order they are found and completed. This approach makes use of Section II's Conditions 1 through 3. It is non-iterative and founded on the "depth" criterion. PSAT, a MATLAB-based toolkit, is used to tackle the OPP optimization problem in [10], [11], and the DeFS approach is contrasted with alternative approaches. In [12], a further DeFS technique is suggested. Despite the DeFS algorithm's computing speed, the solution is not optimal due to the rigid and unitary optimization requirement.
2. Minimum Spanning Tree (MST) The minimum spanning tree (MST) approach is a modified depth first strategy. The MST technique improves the complex and weak convergence of the DeFS approach that also has rapid computing properties. The optimization rule "Find maximum bus network coverage" has been replaced with the rule "On-ramps that link buses and off-ramps that link buses". A large number of simulations have been performed on the Yunnan Power grid in China [13] and the IEEE-14 and IEEE-30 bus systems [12].

### V. METAHEURISTIC METHODS

1. Simulated Annealing (SA): This is achieved by making random changes to the current solution which is a meta-heuristic approach to finding a good solution to an optimization problem. The algorithm is likely to come up with a optimal or near optimal solution if the cooling schedule, or the rate at which the temperature is reduced, is slow. The SA technique suggests a fairly simple objective function that captures the cost of installing and measuring the sensors and their distribution.
2. Genetic Algorithm (GA): Genetic algorithm (GA) is a search heuristic that mimics the process of natural evolution. This heuristic is often used in order to generate successful solutions to search and optimization problems. By using PMU placement criteria, the total number of measurements, the maximum number of measurements received from the repulsive PMU network under different condition criteria such as detecting critical measurements and critical sets not included in the system, the maximum accuracy of estimates, the minimum cost of PMU placement, and the transformation of the network graph
3. Tabu Search (TS) Tabu Search (TS) is a combinatorial search method that uses memory structures to remember previously visited solutions and to explore new solutions for optimization problems. The work presented in [6] provides a novel topological methodology extending the augment incidence matrix and TS algorithm. It can solve the combinatorial OPP

problem with very low computation, and is also very robust. For integer quantities, the method is faster and simpler than standard observability analysis approaches that depend on sophisticated matrix analysis. The TS approach on meter placement has been presented in [16] to maximize topological observability.

4. Particle Swarm Optimization (PSO): An optimization technique called particle swarm optimization (PSO) offers a population-based search process where individuals, referred to as particles, shift places over time. In a multi-dimensional search space, particles flutter around. Each particle uses the optimal place it and its neighbors have found to alter its own position during flight based on its own experience as well as the experiences of nearby particles. The set of particles that surround a particle and its past experiences influence the particle's swarm direction. As an optimization tool, a modified discrete binary version of the particle swarm algorithm (BPSO) is employed in [17] to determine the smallest number of PMUs required for full observability. An enhanced topological observability evaluation based on topological analysis is put into practice by creating a new rule based on analysis of zero-injections. In [18], a BPSO algorithm is presented with the goal of minimizing PMU installation expenses. The cost may vary depending on a number of factors, including the number of nearby branches and the communication conditions at the bus. It has been demonstrated that the latter element is more qualified than traditional approaches.

## VI. CONCLUSIONS

An NP-hard problem is the OPP problem. To address the issue, a variety of optimization strategies have been developed over the past 25 years. The suggested methods fall into three primary groups: metaheuristic, heuristic, and conventional. Researchers can find and employ innovative approaches to solve the difficult OPP problem with the help of the literature study this paper presents.

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